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A Topological Approach to Computer-Aided Sensitivity Analysis

A mathematical development using the theory of linear oriented signal flow graphs proves in a relatively simple way that the sensitivities of any arbitrary system can be calculated using a general purpose digital computer with available software packages for transfer function analysis.

Functional systems consist of interrelated components. (In electronic circuits, for example, the components include elements such as resistors and capacitors that are described by impedance, admittance, and other parameters.) The systems may also be considered as entire subsystems consisting of a number of individual elements connected together as a functioning group and described by a transfer function. The concept of sensitivity remains unchanged, however, regardless of whether impedance, admittance, or transfer functions are used to express subsystem relationships; and this concept may be applied to systems as diverse as sociological systems, where human beings are the subsystem components, and economic systems, where business and production organizations are the subsystem components. The importance of sensitivity as an analytic tool in understanding systems operation is largely due to its ability to show how the variation of an element within the system affects system performance.

One method of measuring system sensitivity involves calculating the partial derivative of some system performance criterion with respect to the variation of a parameter (i.e., a quantitative subsystem characteristic, such as a resistor value). Sensitivity, as thus defined, is a measure of the incremental change in a system performance criterion due to the change in a performance parameter when all other system parameters are held constant. A simple example would be the variation of the gain of an amplifier circuit due to a change in the value of one resistor within the circuit. Further development of the sensitivity concept, from one which provides a measure of the absolute change of a system performance criterion to one which provides the relative change of a performance criterion with respect to the relative change in a system parameter, leads to the derivation of a logarithmic sensitivity which can be expressed as the percentage change in a performance criterion due to a percentage change in the parameter.

The computation of sensitivity is a very tedious operation, prone to error if done manually, especially for large systems; digital computation offers a great advantage, but is performed only with difficulty. Since digital computers do not conveniently differentiate the complex algebraic expressions representing system performance, a systematic procedure must be structured around the more common numerical capabilities of computer operations. Sensitivity and logarithmic sensitivity can be determined systematically by computing certain transfer functions. Since many highly refined transfer-function analysis programs have been developed, it would be valuable if these programs could be used directly in the study of sensitivities.

It has been proven that this is possible by using the theory of signal flow graphs and by developing the topological relationships that express sensitivity. The signal flow graph is ideal, as it illustrates topologically the behavior of a system and the relationship among parameters within the system. The performance criterion in signal flow graph terminology is identical to transmittance, T, which was selected for study. Any transmittance, T, of an arbitrary connected signal flow graph (and the system modeled

(continued overleaf)

by it) can be expressed as a function of the gain of any edge, P, within the graph corresponding to the value of a parameter in the system. By an appropriate differentiation, an expression can be obtained for partial derivatives which permits their computation by transfer functions alone. This expression is

$$\frac{\partial T}{\partial P} = T_1 T_2$$

where T_1 is directly interpretable as the transfer function from the input to the parameter in question. For example, consider any system voltage transfer function where the sensitivity with respect to a resistor in the system is sought. In this case, T_1 is the transconductance from the system input to the resistor; i.e., the resistor current divided by the system voltage which produces that current. T_2 is similarly interpretable as the transfer function from a measurable output of the parameter in question to the output of the system. In this case, T_2 is the ratio of output voltage to the voltage across the resistor. The logarithmic sensitivity can be found simply by multiplying ($\partial T/\partial P$) by P and 1/T (the reciprocal of the network transmittance). This form of computation is satisfactory for practical use in a computer if an algorithm for polynomial arithmetic is available.

Reference:

Munoz, Robert M., and Chan, S. Park: An Optimal Strategy for Topological Analysis of General Networks by Computer. Presented at the Joint Conference on Mathematical and Computer Aids to Design, sponsored by ACM/SIAM/IEEE, October 27-31, 1969, Anaheim Convention Center, Anaheim, California.

Note:

Requests for further information may be directed to:

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Patent status:

No patent action is contemplated by NASA.

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(ARC-10214)